IES/ISS EXAM, 2016

Sl. No. 0005914

A-GSE-P-TUC

STATISTICS

Paper III

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are SIX questions divided under TWO sections.

Candidate has to attempt FIVE questions in all.

All the THREE questions in Section A are compulsory.

Out of the THREE questions in Section B, TWO questions are to be attempted.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

All parts and sub-parts of a question are to be attempted together in the answer book.

Any page or portion of the page left blank in the answer book must be clearly struck off.

Answers must be written in ENGLISH only.

Section – A

All the *three* questions are compulsory.

1. (a) Define Des Raj's ordered estimator for population mean on the basis of a sample of size 2 and show that it is unbiased. 10

(b) Let π_i and π_{ij} $(j \neq i)$ be the inclusion probabilities of first and second order respectively in a simple random sample of size n, selected from a finite population of size N. Then show that

(i) $\sum_{i=1}^{N} \pi_i = n$ and

(ii) $\sum_{(j \neq i)=1}^{N} \pi_{ij} = n(n-1)$. 10

(c) Give a practical example where two-stage sampling scheme may be adopted. For equal size first-stage units, obtain an estimator for population mean in two-stage sampling and its variance. Discuss the problem of allocation of first and second-stage sample sizes for a fixed cost. 20

2. (a) Describe the problem of multicollinearity in general linear model and explain how will you detect it. 10

(b) In usual notation consider the standard linear model : $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{U}$; $\boldsymbol{U} \sim N(0, \sigma^2 \boldsymbol{I})$. Show that the MLE $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ have the distribution $N\left(\boldsymbol{\beta}, \sigma^2 (\boldsymbol{X'X})^{-1}\right)$, assuming $\boldsymbol{X'X}$ to be an invertible matrix. 10

(c) (i) Explain the identification problem in a system of simultaneous equations. State, without proof, the rank and order conditions for identifiability of an equation. 10

(ii) Identify the following system :

$$y_1 = 3y_2 - 2x_1 + x_2 + U_1$$
$$y_2 = y_3 + x_3 + U_2$$
$$y_3 = y_1 - y_2 - 2x_3 + U_3$$
10

3. (a) Construct the price index number for 2010 with 2005 as base year from the following data by using

(i) Laspeyre's

(ii) Paasche's and

(iii) Fisher's method

Item	*Price (in Rs.)*		*Quantity*	
	2010	*2005*	*2010*	*2005*
A	10·50	8·25	6	4
B	6·40	6·00	10	6
C	15·20	10·80	6	5
D	6·25	4·00	8	5

Verify whether the Time Reversal Test is satisfied by the abovementioned index numbers. 10

(b) Define price elasticity of demand (η_p) and interpret when η_p is

(i) <1

(ii) >1 and

(iii) $=1$

If the demand function is $p = 10 - 5x^2$, for what value of x elasticity of demand will be unity ? (x is the quantity demanded and p is the price). 10

(c) Define autocorrelation of lag K of a stationary process. Consider the time series model defined by

$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \alpha_3 X_{t-3} + \in_t$

where $\{\in_t\}$ is white noise.

(i) Show that the autocorrelation coefficient with lag 1 for the process is :

$$\rho_1 = \frac{\alpha_1 + \alpha_2\alpha_3}{1 - \alpha_2 - \alpha_1\alpha_3 - \alpha_3^2}.$$

(ii) Consider the case where

$\alpha_1 = \alpha_2 = \alpha_3 = 0{\cdot}2$.

Comment on the stationarity of this model.

You may use $5 - x - x^2 - x^3 \simeq$

$(1{\cdot}278 - x)\,(3{\cdot}912 + 2{\cdot}278x + x^2)$.

Calculate ρ_1 and ρ_2. 20

Section – B

Attempt any *two* questions.

4. (a) A finite population of size 100 is divided into two strata. In the usual notations, it is given that $N_1 = 60$, $N_2 = 40$, $S_1 = 2S_2$. If a sample of size 24 is to be selected from the population, obtain the number of units to be selected from each of the stratum under Neymann allocation. 10

(b) For the model $y = X\beta + u$ if X and u are correlated show that the OLS estimator for β is not consistent. Discuss the use of instrumental variable technique to obtain a consistent estimator of β. 10

(c) Consider a time series $y_t = T_t + C_t + I_t$ where T_t a trend, C_t a cyclical component and I_t a random component. Discuss the effect of moving averages on cyclical and random components assuming $C_t = a \sin \frac{2\pi t}{\lambda}$. 20

5. (a) Explain Koyck's approach to distributed geometric lag model. 10

(b) What is a chain index number? Discuss its advantages and disadvantages over fixed-base index number. 10

(c) In what sense cluster sampling is different from simple random sampling ? Define an unbiased estimator for population mean in case of cluster sampling with equal cluster size. Compare the efficiency of cluster sampling in terms of intra-class correlation coefficient with respect to simple random sampling without replacement. 20

6. (a) You believe that a set of data is the realization of an MA(1) process $X_n = e_n + \beta e_{n-1}$, where the errors e_n are standard normal. You have calculated the sample auto-covariance function and found that $\hat{\gamma}_0 = 1$ and $\hat{\gamma}_1 = -0{\cdot}25$. Estimate the parameter β. Which value of β do you think you should choose and why ? 10

(b) Describe the Lahiri's method of selecting a probability proportional to size sample from a finite population of size N. 10

(c) Describe lag model and distributed lag model. What are the different lag schemes ? How would you estimate lags by applying ordinary least square ? 20